Observation of chiral surface excitons in a topological insulator Bi$_2$Se$_3$

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The protected electron states at the boundaries or on the surfaces of topological insulators (TIs) have been the subject of intense theoretical and experimental investigations. Such states are enforced by very strong spin–orbit interaction in solids composed of heavy elements. Here, we study the composite particles—chiral excitons—formed by the Coulomb attraction between electrons and holes residing on the surface of an archetypal 3D TI, Bi$_2$Se$_3$. Photoluminescence (PL) emission arising due to recombination of excitons in conventional semiconductors is usually unpolarized because of scattering by phonons and other degrees of freedom during exciton thermalization. On the contrary, we observe almost perfectly polarization-preserving PL emission from chiral excitons. We demonstrate that the chiral excitons can be optically oriented with circularly polarized light in a broad range of excitation energies, even when the latter deviate from the (apparent) optical band gap by hundreds of millielectronvolts, and that the orientation preserves even at room temperature. Based on the dependences of the PL spectra on the energy and polarization of incident photons, we propose that chiral excitons are made from massive holes and massless (Dirac) electrons, both with chiral spin textures enforced by strong spin–orbit coupling. A theoretical model based on this proposal describes quantitatively the experimental observations. The optical orientation of composite particles, the chiral excitons, emerges as a general result of strong spin–orbit coupling in a 2D electron system. Our findings can potentially expand applications of TIs in photonics and optoelectronics.

Significance

We observe composite particles—chiral excitons—residing on the surface of a topological insulator (TI), Bi$_2$Se$_3$. Unlike other known excitons composed of massive quasiparticles, chiral excitons are the bound states of surface massless electrons and surface massive holes, both subject to strong spin–orbit coupling which locks their spins and momenta into chiral textures. Due to this unusual feature, chiral excitons emit circularly polarized secondary light (photoluminescence) that conserves the polarization of incident light. This means that the out-of-plane angular momentum of a chiral exciton is preserved against scattering events during thermalization, thus enabling optical orientation of carriers even at room temperature. The discovery of chiral excitons adds to the potential of TIs as a platform for photonics and optoelectronics devices.

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Experimental Results

Polarized PL. Fig. 1C depicts the intensities of right- and left-circularly polarized PL signals excited by right-circularly polarized light, $I_{RR} (\omega, T)$ and $I_{RL} (\omega, T)$, respectively, where $\omega$ is the energy of emitted photons and $T$ is temperature. Two emission peaks at about 1.5 eV and 2.3 eV in the visible range behave in strikingly different ways when excited by circularly polarized light. Namely, the peak at 1.5 eV is unpolarized; i.e., emission of right- and left-circularly polarized light has the same intensity. The peak at 2.3 eV behaves as an ordinary PL signal observed in conventional semiconductors, where the memory about the incident photon polarization is lost during thermalization of optically generated electron–hole pairs. In contrast, the peak at 2.3 eV is almost fully polarized with the same polarization as the excitation photon; i.e., emission occurs in the RR channel but not in the RL one.

We note that excitons are not usually observed in semimetals and doped semiconductors, with the Fermi level crossing the conduction band, because the exciton state is likely to be hybridized with the conduction band states. Even if the exciton level remains within the gap, the optically produced electron–hole pairs would relax rapidly to the Fermi energy in a non-radiative way. In our case, RSS and SS2 are gapped from the Fermi level, and thus the electron–hole bound state can decay radiatively, resulting in observed PL.

To further elucidate the nature of polarized PL in Bi$_2$Se$_3$, we compare the spectra measured in different polarization geometries. The results are reproducible in the “time-reversed” geometry; i.e., the polarized PL signals with right- (Fig. 2A) and left-circularly polarized excitation (Fig. 2B) show the same line shape and intensity. This suggests that the light-emitting states are doubly degenerate, with components amenable to independent excitation by right- or left-circularly-polarized photons.

If relaxation processes of these states preserve their angular momenta, the secondary photons are emitted with the same polarization as that of the excitation one. For reasons that will become clear later on, we denote these states as $|J_z = 1\rangle$ and $|J_z = -1\rangle$ (Fig. 1D).

In Fig. 2 C–F we show intensity of PL excited with linearly polarized light, with X (Y) denoting linear polarization parallel (orthogonal) to the plane of incidence. We find that the PL signal has almost the same intensity and line shape in both circular polarization channels, regardless of whether the excitation photon is X or Y polarized (Fig. 2 C–E). This suggests that a linearly polarized photon, being decomposed into right- and left-circularly polarized ones, can independently excite both the $|J_z = 1\rangle$ and $|J_z = -1\rangle$ states. That linear polarization is not preserved in the PL process (Fig. 2F) and that $I_{RL}(\omega, T)$ coincides with $|I_{LL}(\omega, T) + I_{RL}(\omega, T)|/2$ (Fig. 2C) imply that quantum coherence is not preserved during the relaxation of electron–hole pairs. As a result, the $|J_z = 1\rangle$ and $|J_z = -1\rangle$ excitonic states act as two independent emitters, which preserve linear but not circular polarization. This property of emission from Bi$_2$Se$_3$ surface states is in contrast to polarized PL observed in TMD monolayers, where both circular and linear polarizations are preserved due to valley quantum coherence (13, 14).

Dependence on the Energy of Incident Photons. Such a high degree of circular polarization for PL cannot originate from the bulk bands, which are spin degenerate (29). However, all three surface bands in Fig. 1A and B exhibit spin-momentum locking and could lead to optical orientation of spins with circularly polarized light. To identify the electron bands responsible for polarized PL, we study the excitation dependence of the peak intensity. Fig. 3 depicts the intensity of polarized PL measured with a
right-circular excitation with six different energies, denoted by the arrows in each panel. As one can see from Fig. 3, the polarized PL peak, whose position is marked by the dashed line, is absent for excitation energies below 2.4 eV. This implies that the electron and hole bands involved in forming the exciton are separated by at least 2.4 eV. By comparing the PL spectra in the top two panels of Fig. 3, we note, however, that the spectrum for the excitation energy of 2.38 eV does not exhibit any visible features at the exciton energy, i.e., at 2.3 eV, whereas weak PL for the excitation energy of 2.33 eV is enhanced at 2.3 eV. Also, the enhancement occurs primarily in the polarized (RR) channel, whereas PL at the excitation energy of 2.38 eV is not polarized. We argue that this reentrant behavior is an indication of the resonant excitation of dipole-allowed exciton states (30).

Despite the overall intensity, the difference between $I_{RL}(\omega)$ and $I_{RR}(\omega)$, depicted in Fig. 3 by the light and dark colored lines, respectively, also changes with the excitation energy. In Fig. 4 we show the integrated PL intensity difference, $\int_2^{2.6} (I_{RR}(\omega, T) - I_{RL}(\omega, T)) d\omega$, vs. the excitation energy $\omega_0$. The polarized PL peak is observed only for excitation energies between 2.6 eV and 3.0 eV. Comparing the excitation profile with the known band structure of Bi$_2$Se$_3$ (SI Appendix, section 2), we conclude that the only possible interband transition is from RSS to SS2 bands.

**Depolarization.** To analyze polarization-preserving PL quantitatively, we decompose $I_{RR}(\omega, T)$ and $I_{RL}(\omega, T)$ into two spectral contributions (Fig. 3): (i) a broad unpolarized emission band, $f(\omega, T)$, and (ii) a narrower peak that is almost fully polarized, with intensity defined as $L_R(\omega, T) = I_{RR}(\omega, T) - f(\omega, T)$. We note that $f(\omega, T)$ and $L_R(\omega, T)$ have distinct lineshapes and therefore are likely to have different origins. We henceforth focus on the polarized PL signal, $L_R(\omega, T)$. A small fraction of $L_R(\omega, T)$ is also present in the orthogonal polarization emission, $L_L(\omega, T) = I_{RL}(\omega, T) - f(\omega, T) = r(\omega) L_R(\omega, T)$, where $r(\omega) = I_{RR}(\omega, T) / I_{RR}(\omega, T) - f(\omega, T)$ is the depolarization ratio (Material and Methods).

In Fig. 4, we plot the temperature dependence of $L_R(\omega, T)$, excited with 2.6-eV right-circularly polarized light. While PL is much stronger at 15 K, emission remains polarized even at 300 K with the same $r(\omega) \approx 0.1$ for all temperatures (SI Appendix, Fig. S6). This demonstrates that while heating shortens the exciton lifetime, it has little impact on polarization of the exciton emission.

Assuming that the depolarization process occurs mostly during the energy relaxation of the electron and hole within the corresponding bands, we expect $r(\omega)$ to approach the direct surface band gap. Fig. 1F shows that $r(\omega)$ linearly extrapolates to zero at $\omega_0 \approx 2.5$ eV, which suggests that the band gap should be close to this value. This is consistent with a direct transition between the top branches of RSS and SS2, shown by the blue arrow in Fig. 1A.
Given atomically smooth and freshly cleaved surfaces, which are realized in Bi$_2$Se$_3$. An interacting electron–hole pair is described by a 4 × 4 two-body Hamiltonian

$$H_{ab}(p, k) = \text{HSS}_2 \left( \frac{p + k}{2} \right) \otimes 1_{\sigma_h} - 1_{\sigma_e} \otimes \text{HSS}_2 \left( -\frac{p + k}{2} \right)$$

$$+ 1_{\sigma_e} \otimes 1_{\sigma_h} V(r),$$

where $r = r_e - r_h$ is the relative position of the electron and hole, $V(r)$ describes the Coulomb interaction, $p = -i \nabla r$, and $k$ is the momentum conjugate to $(1/2) (r_e + r_h)$. For $k = 0$, the eigenvalues of $H_{ab}(p, k)$ have a W-shaped dispersion resembling a multilayer Mexican hat (compare Fig. 1D and SI Appendix, Fig. S2). If both electron and hole bands were massless, a bound state would not be possible. However, the two-body bands originating from $H_{ab}(p, 0)$ are bounded from below for any values of $n$ and $\alpha$ by the $p^2$ term in the RSS band. Therefore, the Coulomb attraction between electrons and holes lead to excitonic bound states.

### Eigenstates and Optical Transitions

In what follows, we focus on the case of zero total momentum ($k = 0$) appropriate for direct optical transitions studied in this report. If $V(r)$ is axially symmetric, the $z$ component of the angular momentum of an electron–hole pair

$$J_z = 1_{\sigma_e} \otimes 1_{\sigma_h} (i \partial_{\phi} \sigma^z_e) + \frac{1}{2} 1_{\sigma_e} \otimes \sigma^z_h + \frac{1}{2} \sigma^z_e \otimes 1_{\sigma_h}$$

is a good quantum number although neither the orbital angular momentum nor spin is a good quantum number on its own. (Here, $\phi$ is the azimuthal angle of $p$.) Therefore, the eigenstates of Eq. 2 can be classified by $J_z$. The Schrödinger equation defined by the Hamiltonian in Eq. 2 can be solved by the following ansatz for the four-component spinor wavefunction in the momentum-space representation (SI Appendix, section 1B),

$$\psi(p) = e^{i J_z \phi} \begin{pmatrix} \psi_1(p) e^{-i \phi} \\ \psi_2(p) \\ \psi_3(p) \\ \psi_4(p) e^{i \phi} \end{pmatrix},$$

where $p \equiv |p|$. To understand the general properties of the resulting discrete states and, in particular, their spin structure, it is instructive to replace the interaction potential by a model short-range attraction $V(r) = -\lambda \delta(r)$, which provides a reasonable approximation for Coulomb interaction screened by free carriers. In this case, algebraic equations for amplitudes $\psi_1(p), \ldots, \psi_4(p)$ have nontrivial solutions for an infinitesimally small $\lambda$, but only for states with $J_z = 0$ and $J_z = \pm 1$ (SI Appendix, section 1A). The bound states with $J_z = \pm 1$, labelled as $|J_z = \pm 1\rangle$ in Fig. 1D, are doubly degenerate, whereas the two states with $J_z = 0$, labeled as $|J_z = 0\rangle$, have different energies$^1$. Within the backscattering geometry of our experiment, circularly polarized light can produce only excitations with $\Delta J_z = \pm 1$. Assuming no cross-relaxation between $|J_z = \pm 1\rangle$ and $|J_z = 0\rangle$ states, we expect a single PL peak arising from recombination of the $|J_z = \pm 1\rangle$ exciton, which is consistent with the data (Fig. 1C).

The above argument is suitable for explaining polarized PL excited by photons with energies close to the Mexican-hat minimum of Fig. 1D. One could expect that scattering by phonons

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$^1$Strictly speaking, exciton states have to be classified within the surface symmetry group, which is $G_{\sigma} = $ for actual Bi$_2$Se$_3$ (33) or $G_{\sigma} = $ for a rotationally invariant Hamiltonian in Eq. 2. Inspecting the exciton wave functions in Eq. 4, we find that the two $|J_z = 0\rangle$ states are fully symmetric with respect to all symmetry operators of both $G_{\sigma}$ and $G_{\sigma} = $ groups and therefore belong to the $A_\sigma$ irreducible representation. On the other hand, the doubly degenerate $|J_z = \pm 1\rangle$ states belong to the $E_\sigma$ representation, which transform as an in-plane electric dipole.

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Theoretical Model

### Surface States

Now we turn to the interpretation of the experimental results. In general, polarization-preserving PL is possible only if the spin degeneracy of electron states is lifted by breaking either time-reversal or inversion symmetries. Since a bulk Bi$_2$Se$_3$ crystal is nonmagnetic and centrosymmetric, we argue that observed polarized PL must be entirely due to surface bands.

In the following, we build a minimal model that explains all of the key aspects of the experimental observations, by considering optically excited electrons and holes in SS2 and RSS bands, respectively. Based on the first-principle calculation of the electronic band structure (Fig. 1B and SI Appendix, Fig. S4), we assert that both SS2 and RSS states correspond to $J_z = \pm 1/2$ projections of the total angular momentum on the $z$ axis and thus can be described by $2 \times 2$ Pauli matrices. Also, the mass term in SS2 is by more than a factor of 4 smaller than the corresponding term in RSS and thus can be neglected (SI Appendix, section 2). With these assumptions, we use the Hamiltonians

$$
H_{SS2}(p) = \Delta 1_{\sigma_e} + v(\sigma_e \times p) \cdot \hat{z}, \\
H_{RSS}(p) = -\frac{p^2}{2m_e} 1_{\sigma_h} - \alpha(\sigma_h \times p) \cdot \hat{z}
$$

[1]

to describe the massless Dirac electrons near the SS2 touching point (31) and the massive Rashba holes near the RSS touching point (32), respectively. Here, $\Delta$ is the energy difference between the Dirac points of RSS and SS2, $v$ is the Dirac velocity, $m_e > 0$ is the effective hole mass, $\alpha$ is the Rashba coefficient, $\hat{z}$ is a unit vector normal to the surface, $\sigma_e$ and $\sigma_h$ are the vectors of Pauli matrices in the SS2 and RSS spin subspaces, respectively, and $1_{\sigma_e}$ and $1_{\sigma_h}$ are the identity matrices in the same subspaces. The linear-in-$p$ terms describe the effect of SOC which locks electron spin at $90^\circ$ to its momentum. We note that although the RSS band is not topologically protected, in contrast to the SS1 and SS2 bands, its band parameters are still expected to be universal
couples the \( | J_z = +1 \rangle \) and \( | J_z = -1 \rangle \) states for energies above the minimum, which would cause an increase of \( r(T) \) with \( \omega_0 \). However, we see only a moderate increase of \( r(T) \) even if \( \omega_0 \) is about 300 meV above the Mexican-hat minimum (Fig. 3). To explain the preservation of optical orientation during energy relaxation, we note that a transition between the \( | J_u = \pm 1 \rangle \) states do not conserve the \( z \) component of the angular momentum \( J_z \) would require scattering by nonsymmetric bosons or by a magnetic impurity. It is known that nonsymmetric surface phonons in \( Bi_2Se_3 \) are weak (34, 35), hence \( | J_u \rangle \) remains approximately conserved during the energy relaxation. It would be interesting to study in the future the interaction between the chiral exciton and other more exotic collective modes, such as the Dirac plasmons and chiral spin modes (27, 36, 37). Importantly, the \( | J_z = \pm 1 \rangle \) exciton states can also be resonantly populated with circularly polarized 2.3-eV excitation (Fig. 3), which suggests that the exciton states are dipole allowed and thus confirms the proposed model.

**Bound-State Energies.** The theoretical model described above allows one to extract quantitative characteristics of the exciton spectra. The exciton energies for a short-range interaction as functions of the dimensionless coupling constant \( u = m_0 \lambda/2\pi \hbar^2 \) are shown in Fig. 5. With the band parameters extracted from ARPES data (17, 18), one finds for the absorption edge in the \( | J_z = \pm 1 \rangle \) channel \( E_{\text{ex}} = \Delta - m_0 (\alpha + v) \sqrt{2}/2 \approx 1.8 \) eV, which is somewhat smaller but comparable to the observed value of 2.48 eV. For a more realistic case of the Coulomb interaction (which is assumed to be weak), the binding energy can be estimated as \( u^{\pm 1} = E_{\text{ex}}^{\pm 1} = -4 \rho^* \ln^2 [2m_0 (\alpha + v) ab/e^2 \hbar] \) (38, 39), where \( \rho^* = m_0 e^4/[2\hbar^2 \varepsilon_0] \approx 0.02 \) eV is the effective Rydberg, \( \varepsilon_0 \approx 13 \) is the effective dielectric constant of seminfinite \( Bi_2Se_3 \) for frequencies above the topmost phonon mode, \( ab = \hbar^2 \varepsilon_0 / m_0 e^2 \) is the effective Bohr radius, and \( e = 2.718 \ldots \) is the base of the natural logarithm. The (large) logarithmic factor in the bound-state energy arises because massive holes with energies close to the minimum of the Rashba spectrum exhibit an effectively 1D motion (38). In 1D, the bound-state energy in a weak potential \( U(x) \) is proportional to \( \int dx U(x)^2 \) (hence the \( \ln^2 \) factor for the \( 1/x \) potential). A more accurate result can be obtained by numerical solution of the Schrödinger equation (39) which gives 0.22 eV for the bound-state energy, whereas the observed value is 0.2 eV. We thus conclude that our theoretical model is in quantitative agreement with the data.

We note that while \( \alpha \) and \( v \) may vary slightly from sample to sample, the chiral exciton energy depends only logarithmically on the band structure parameters, at least as long as the Coulomb attraction between electron and hole is sufficiently weak. Furthermore, it is known from ARPES measurements, nonlinear optics, and first-principle calculations that, while the position of the Fermi level is very sensitive to surface preparation, the surface states are rather robust against nonmagnetic dopants (21, 41, 42). In our case, the surface states composing the chiral exciton are far away from the Fermi level and thus should be even less sensitive to surface contamination. This naturally explains the reproducibility of the observed features between samples.

**Conclusions.** We used polarization-resolved PL spectroscopy to study the secondary emission from the surface states of an archetypical topological insulator \( Bi_2Se_3 \). When the crystal is excited with 2.5–2.8 eV circularly polarized light, we detect emission of the same polarization at 2.3 eV. Polarization of emitted light is preserved even if the excitation energy is hundreds of millielectronvolts above the emission threshold energy. We assign such emission as resulting from recombination of exciton states: chiral excitons. We propose that chiral excitons are made of (topologically protected) massless electrons and massive holes, both residing on the surface of \( Bi_2Se_3 \) and characterized by chiral spin textures. The exciton states can be characterized by the eigenvalues of the out-of-plane total angular momentum, \( J_z \). Based on the results of our theoretical model, we identify the doublet of degenerate states with \( J_z = \pm 1 \) as being responsible for observed polarization-preserving PL. The most surprising finding is that polarization of chiral exciton PL is preserved up to room temperature and robust with respect to chemical substitution, which we attribute to the weakness of spin-flip scattering between surface states with opposite helicity. In this way, chiral excitons are fundamentally different from other known excitons that also preserve helicity (4, 13). Controlled optical orientation of chiral surface excitons may facilitate new photonics and optoelectronics applications of topological insulators.

**Materials and Methods.**

**Material Growth.** All data presented in the main text are collected from bulk single crystals grown by a modified Bridgman method. Mixtures of high-purity bismuth (99.999%) and selenium (99.999%) with the mole ratio Bi:Se = 2:3 were heated to 870 °C in sealed vacuum quartz tubes for 10 h and then slowly cooled to 200 °C at the rate of 3 °C/h, followed by furnace cooling to room temperature.

**Experimental Setup.** The crystals were cleaved before cooldown in a glove bag filled with nitrogen gas and were transferred into a continuous-flow liquid helium optical cryostat without exposure to atmosphere. A solid-state laser was used for 2.33-eV (532 nm) excitation, a diode laser was used for 2.77-eV (447 nm) excitation, and a Kr+ ion laser was used for all other excitations, with laser spot size roughly 50 × 50 μm². The power density on the sample is kept below 0.7 kW/cm², and all temperatures shown were corrected for laser heating with 1 K/mW. The polarized secondary emission was analyzed and collected by a custom triple-grating spectrometer with a liquid nitrogen-cooled CCD detector.

The intensity \( I_p (\omega, T) \) was corrected for the laser power and spectral response of the spectrometer and CCD, where \( I_p (\nu) \) denotes the direction of incident (collected) photon polarization, \( \omega \) is energy, and \( T \) is temperature.
The scattering geometries used in this experiment are denoted as $\mu\nu=Rr$, $Rt$, $XX$, and $XY$. $R=+X+Y$ and $L=X-Y$ denote the right- and left-circular polarizations, respectively, where $Y$ (or $X$) denotes linear polarization parallel (orthogonal) to the plane of incidence. Here, we follow the “spectroscopy convention” for the “handedness” of circularly polarized light. That is, the right and left polarizations refer to the angular momentum measured in the laboratory frame, rather than to the helicity of the photon.

**Photoluminescence Background Subtraction.** With right-circularly polarized excitation, the measured PL intensities can be decomposed into two parts,

\[
I_R(\omega, T) = I_R(\omega, T) + f(\omega, T) I_L(\omega, T) + f(\omega, T),
\]

where $I_R(\omega, T)$ and $I_L(\omega, T)$ denote right and left circularly polarized PL, respectively, and $f(\omega, T)$ features the unpolaredized broad background. We assume an energy independent depolarization ratio $r(\omega, T) = \frac{c_2(\omega, T)}{c_1(\omega, T)}$. Inserting $r(\omega, T)$ into the above expression of $I_R(\omega, T)$, we can write the unpolarized emission as

\[
f(\omega, T) = \frac{I_R(\omega, T) - r(\omega, T) - I_L(\omega, T)}{1 - r(\omega, T)}.
\]

Then, $r(\omega, T)$ is determined by minimizing sharp features in $f(\omega, T)$ around the 2.3-eV PL peak. The circularly polarized PL can be calculated knowing $r(\omega, T)$,

\[
l(\omega, T) = \frac{I_R(\omega, T) - I_L(\omega, T)}{1 \pm r(\omega, T)}.
\]

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